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Phonon assisted tunnelling through double barriers

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Abstract. Tunnelling through double barriers with temporal oscillation of barrier heights is solved by a time-dependent two-potential formula. We show that the resonant tunnelling survives in the extended phonon modes and that the coherent aspect of phonon assisted tunnelling is important. The relevance to certain experiments is also discussed.

1. Introduction

Tunnelling through a barrier is a characteristic and fundamental phenomenon in quantum mechanics. The history of tunnelling studies can be traced back to the 1930s when quantum mechanics just emerged. Nowadays, tunnelling still receives a lot of attention because of its potential applications in the new generation of high speed and high quality electronic devices from novel transistors to lasers and detectors (Capasso *et al* 1986). Particularly, the ongoing research focuses on the aspects of resonant tunnelling through barriers (Tsu and Esaki 1973, Chang *et al* 1974, Ricco and Azbel 1984) and interactions with a phonon bath (Calderia and Legget 1981, 1983, Bialek *et al* 1986). The advent and impressive progress of molecular beam epitaxy (MBE) and vapour phase epitaxy (VPE) has made it possible to fabricate semiconductor devices containing ultrathin ($\leqslant$100 Å) layers. For example, double barrier structures can be formed by sandwiching a thin GaAs layer ($<$100 Å) between two GaAlAs barriers. A MOSFET is another example of the double barriers which possesses many interesting properties. Since the layer thickness falls into the range of the de Broglie wavelength of electrons on a Fermi surface and the typical barrier heights are of the order of several tenths of eV, the quasi energy levels in the well between two barriers are discretised. The same situation occurs in the GaAs–AlAs superlattice or so-called 'quantum wells'. When the Fermi energy matches with these quasi energy levels (which can be controlled by an applied voltage bias), a sharp maximum of conductance occurs. This resonance phenomenon manifests itself as peaks or humps in the tunnelling current versus voltage plot, i.e. the characteristic *IV* curve. Here, the peak-to-valley ratio of the current is the measure of performance of a high quality device (Sollner *et al* 1984, Tsuchiya and Sakaki 1986). Qualitatively, the resonant tunnelling is well understood (Ricco and Azbel 1984): a large electronic density builds up near the quasi levels between the barriers because the wave leaking through the first barrier is constructively interfering with the reflections off the second barrier, thus, the large tunnelling current through the second barrier is achieved. This picture of resonant

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tunnelling remains valid for sequential tunnelling in quantum wells and even in disordered materials where the quasi localised states exist as a result of Anderson localisation.

Among all the factors complicating the analysis of the tunnelling current, electron–phonon scattering is the dominant process. While the time scale of scattering (either elastic or inelastic) sets up the width of the resonance (Stone and Lee 1986), phonon assisted tunnelling leads to the sideband around the principal peak. Nevertheless, the phonon effect is extremely difficult to observe in a single-barrier case since it is masked by the much larger elastic tunnelling (Goldman *et al* 1987), whereas in the double-barrier case, there is experimental evidence (Goldman *et al* 1987) as well as theoretical calculations (Wingreen *et al* 1988, Stone *et al* 1985) showing the side peaks around the resonant peak. As far as theory is concerned, there are two approaches in the literature, the difference of which lies in the description of the phonon field. Wingreen *et al* (1988) take a complete quantum treatment of phonon modes whereas Stone *et al* (1985) describe them as an external time-dependent field. The latter treatments resemble the semi-classical theory of photon–matter interaction. However, Stone *et al* (1985) only deal with the local phonon mode which is mathematically taken as a spatial δ function and is located inside the well. This work considers a different model: the phonon modes are extended in the region of the two barriers. We will show later that resonance tunnelling also survives in this model. Furthermore, the relative phase between two extended spatial modes will lead to either constructive or destructive interference of the transmission current. We will discuss this novel feature and the relevance to experiments. From the technical point of view, the solutions of the time-dependent Schrödinger equation with a harmonic oscillating potential are obtained by Stone *et al* (1985) and Büttiker and Landauer (1982) using an iteration method which goes from zeroth-order to higher-order side bands. We adopt a different route because of the fast growing complexity associated with more barriers and higher-order phonon terms. This new method views tunnelling as a scattering event and assumes the static barrier problem is solvable and is taken as an unperturbed state. The harmonic oscillation part of the potential is added as the perturbation. Using the time-dependent version of the two-potential formula, we solve the phonon assisted tunnelling in the double barrier and its coherent aspect.

2. Phonon assisted tunnelling in double barriers

In order to answer the important question of the role of a temporal oscillation of barriers in resonant tunnelling, Stone, Azbel and Lee (SAL) proposed a double square barrier potential with a time-dependent perturbation inside the well (Stone *et al* 1985). This potential can be written as

$$V(x, t) = u_1(x) + u_2(x) + 2\gamma\delta(x)\cos\omega t. \tag{1}$$

They concluded that resonant tunnelling survives in the localised oscillatory potential as shown in (1). Since this potential can only simulate some localised phonon modes in GaAs thin layers (or oxide layers in MOSFETS), one would like to generalise it to the

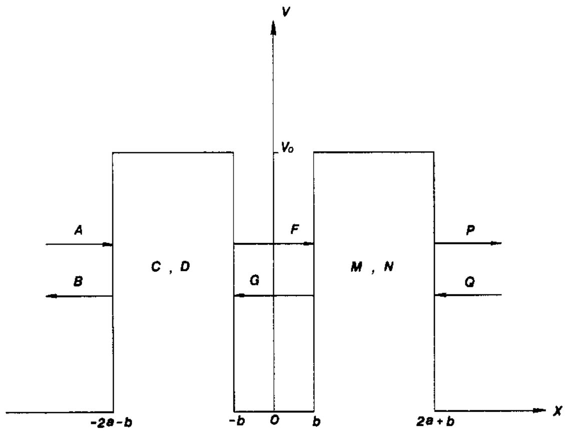

Figure 1. The schematic diagram for a double-barrier potential. The ten coefficients for electronic wavefunctions in three regions are labeled.

extended phonon modes. This article considers a different model—one can think of it as a double barrier version of Büttiker and Landauer's (BL, 1982) generic model:

$$V(x,t) = \begin{cases} V_0 + V_1 \cos \omega t & \text{if } -2a-b<x<-b \\ 0 & \text{if } -b<x<b \\ V_0 + V_1 \cos(\omega t + \alpha) & \text{if } b<x<2a+b. \end{cases} \tag{2}$$

The potential $V(x, t)$ is sketched in figure 1. The interesting feature of this model is the relative phase difference α, which can represent the coherent lattice motion if α is a fixed phase, or an incoherent motion if α is a random variable like for the case of a thermal phonon bath. We will reveal later that α can lead to interference effects if it remains fixed. Let us first rewrite the potential in (2)

$$V(x,t) = \tilde{V}_0(x) + \tilde{V}_1(x,t) \tag{3a}$$

where

$$\tilde{V}_0(x) = \begin{cases} V_0 & \text{if } -2a-b<x<-b \\ 0 & \text{if } -b<x<b \\ V_0 & \text{if } b<x<2a+b \end{cases}$$

and

$$\tilde{V}_1(x,t) = \tilde{V}_1^+(x)\exp(\mathrm{i}\omega t) + \tilde{V}_1^-(x)\exp(-\mathrm{i}\omega t) \tag{3b}$$

where

$$\tilde{V}_1^{\pm}(x) = \begin{cases} \tfrac{1}{2}V_1 & \text{if } -2a-b<x<-b \\ 0 & \text{if } -b<x<b \\ \tfrac{1}{2}V_1 \exp(\pm \mathrm{i}\alpha) & \text{if } b<x<2a+b. \end{cases}$$

Similar to the well known photoabsorption case, the term $\tilde{V}_1^- \exp(-\mathrm{i}\omega t)$ is responsible for the absorption of a quantum in the rotating wave approximation and the term $\tilde{V}_1^+ \exp(\mathrm{i}\omega t)$ is responsible for the emission of a quantum.

We then define the following coefficients for the scattering wavefunction in different regions, A, F and P are three coefficients of rightward propagating waves in three corresponding regions; B, G and Q are their counterparts for leftward ones; C and M stand for amplitudes of attenuating waves $\exp(-\kappa x)$; and D and N, ones for $\exp(\kappa x)$. The ten coefficients are also illustrated in figure 1.

It is usual in time-dependent potential problems to use the standard procedure (Messiah 1961) to derive the following two-potential formula (Jiang 1989) for the one-quantum absorption, scattering amplitude in 1D† case

$$T_1 = (\mathrm{i}m/2\hbar^2 k)\langle\Psi_{k_1}^{(-)}|\tilde{V}_1^-(x)|\Psi_k^{(+)}\rangle \tag{4}$$

where $\Psi_{k_1}^{(-)}$ is an incoming scattering wave by the static potential $V_0(x)$, and $\Psi_k^{(+)}$ is an outgoing wave by the total time-dependent potential $V(x,t)$.

It can be shown that the conventional time-independent two-potential formula (Taylor 1972) can be recovered if the potential is static.

In practice, (4) can be further simplified by taking $\Psi_k^{(+)}$ as a scattering state under V_0 with the assumption that V_1 is small such that $\Psi_k^{(+)}$ is not much different from its stationary counterpart. This is similar to the distorted wave Born approximation (DWBA) and is used throughout this work. Hence, both $\Psi_{k_1}^{(-)}$ and $\Psi_k^{(+)}$ are stationary eigenstates under a static potential V_0, except that they have slightly different energies determined by the kinetic relation $k_1^2 = k^2 + 2m\omega/\hbar$.

The continuity of the wavefunction and its logarithmic derivative leads to the transfer matrix relations (Merzbacher 1970) connecting (A,B), (C,D), (F,G), (M,N), and (P,Q). Using these relations, it is trivial to calculate the scattering states with two different boundary conditions: (i) outgoing state $\Psi_k^{(+)}$, i.e. $A=1$, $Q=0$; (ii) incoming state $\Psi_{k_1}^{(-)}$, i.e. $B=0$, $P=1$.

The transmission amplitude $T_{\mathrm{double}} = P/A$ is thus given by

$$T_{\mathrm{double}} = \exp(-4\mathrm{i}ka)/[(\cos 2\kappa a + (\mathrm{i}\varepsilon/2)\sinh 2\kappa a)^2 + (\eta^2/4)\sinh^2 2\kappa a \exp(4\mathrm{i}kb)] \tag{5}$$

where

$$\varepsilon = \kappa/k - k/\kappa \qquad \eta = \kappa/k + k/\kappa.$$

Using (4), the one-phonon absorption tunnelling amplitude is given by

$$\begin{aligned} T_1 = \frac{\mathrm{i}m}{2\hbar^2 k}\bigg(&\int_{-2a-b}^{-b} \mathrm{d}x\,(C_1^* \exp(-\kappa_1 x) + D_1^* \exp(\kappa_1 x))V_1(C\exp(-\kappa x) + D\exp(\kappa x)) \\ &+ \int_b^{2a+b} \mathrm{d}x\,(M_1^* \exp(-\kappa_1 x) + N_1^* \exp(\kappa_1 x))V_1 \exp(-\mathrm{i}\alpha) \\ &\times (M\exp(-\kappa x) + N\exp(\kappa x) + N\exp(\kappa x))\bigg). \end{aligned} \tag{6}$$

The calculation can be simplified if only the dominant exponential terms are retained, e.g. C_1 is of the order of $\exp[-\kappa_1(6a+b)]$, therefore, the terms including CC_1^* and

† In the 3D case, $T_1 = (m/4\pi\hbar^2)\langle\Psi_{k_1}^{(-)}(r)|V_1(r)|\Psi_k^{(+)}(r)\rangle$.

DC_1^* are negligible in comparison to the other terms in the first integration (6). Furthermore, the low-ω approximation is used

$$\hbar\omega \ll \hbar^2 k^2/2m \qquad \hbar\omega \ll \hbar^2 \kappa^2/2m \tag{7}$$

such that $\kappa - \kappa_1 \ll \kappa + \kappa_1$. In this case, the CD_1^* term is much larger than the DD_1^* term. These approximations are also in accordance with BL's treatment. Thus, the first part in (6) is

$$T_1^{(1)} \exp(2ik_1 a) = T\exp(2ika)(V_1/2\hbar\omega)(\exp(\omega\tau) - 1)\exp(\omega\tau)\exp(i\omega\tau'/2). \tag{8}$$

where

$$k_1 = k + m\omega/\hbar k \qquad \kappa_1 = \kappa - m\omega/\hbar\kappa \qquad \tau = 2ma/\hbar\kappa \qquad \tau' = 2mb/\hbar k.$$

Among the contributions in the second part of (6), the MN_1^* term is the dominant one, therefore

$$T_1^{(2)} = T\exp(2ika)(V_1/2\hbar\omega)(\exp(\omega\tau) - 1)\exp(-i\omega\tau'/2)\exp(-i\alpha). \tag{9}$$

Combining (8) and (9), we obtain

$$T_1 \exp(2ik_1 a) = T\exp(2ika)(V_1/2\hbar\omega)(\exp(\omega\tau) - 1) \times \{\exp(\omega\tau) + \exp[-i(\omega\tau' + \alpha)]\}\exp(i\omega\tau'/2). \tag{10}$$

Equation (10) is the central result of this work. Its validity can be checked by the case $b = 0$, (therefore, $\tau' = 0$) and $\alpha = 0$. It does lead to BL's single-barrier result (Büttiker and Laudauer 1982), i.e.

$$T_1 \exp(2ik_1 a) = T\exp(2ika)(V_1/2\hbar\omega)(\exp(2\omega\tau) - 1). \tag{11}$$

Equation (10) clearly shows the coherence effect of the time-dependent motion of two barriers. For the simplest case ($b = 0$) which is a single-barrier formed by two adjacent barriers, $\alpha = 0$ gives the constructive interference with a factor $(\exp(2\omega\tau) - 1)$ in the magnitude of T_1, and $\alpha = \pi$ gives the destructive interference with a factor $(\exp(\omega\tau) - 1)^2$. Now, if the motions of the two adjacent barriers are uncorrelated, i.e. α is a stochastic variable, the transmission current is the summation of two currents enhanced by the two barriers respectively. In this case, the first current in (8) will carry a factor $\exp(2\omega\tau)(\exp(\omega\tau) - 1)^2$ and the second one in (9) will carry a factor $(\exp(\omega\tau) - 1)^2$. Thus, the total current at energy $E_1 = \hbar^2 k_1^2/2m$ is

$$|T_1|^2 \hbar k_1 = |T|^2 (V_1/2\hbar\omega)^2 \hbar k_1 (\exp(\omega\tau) - 1)^2 (\exp(2\omega\tau) + 1). \tag{12}$$

The importance of interference due to coherent lattice motion can be measured by

$$(|T_1^{\text{constructive}}|^2 - |T_1^{\text{incoherent}}|^2)/|T_1^{\text{incoherent}}|^2 = 2/(\exp(\omega\tau) + \exp(-\omega\tau)). \tag{13}$$

Usually, $\exp(\omega\tau)$ is of the order of 1 and in a model system of a double barrier with ultrathin well-material such that $b = 0$, if $2a = 80$ Å, $V_0 = 400$ meV, $E = 185$ meV, $\omega = 300$ K, it leads to $\omega\tau = 2.2$ and the measure of the coherence is about 12%. However, in the case $2b = 50$ Å, as shown later in the numerical calculation, the coherence effect can count for up to a 60% contribution.

We now proceed to the resonance of double-barrier tunnelling. The transmission coefficient through the double barriers is shown in (5). When $b = 0$, it leads to a single-barrier result

$$T_{\text{single}} = \exp(-4\,\mathrm{i}ka)/(\cosh 4\kappa a + (\mathrm{i}\varepsilon/2)\sinh 4\kappa a). \tag{14}$$

Notice that now the barrier width is $4a$. The magnitude of T_{single} is a monotonic increasing function of k. The case $b \neq 0$ represents a double barrier. First, we examine its static resonant tunnelling. Equation 5 shows that the transmission amplitude T_{double} is a periodic function of the width of the well with a period equal to π/k, i.e. when $2b' = 2b + n\pi/k$, the well with width $2b'$ gives the same T_{double} as the well with width $2b$. Secondly, T_{double} is not a monotonically increasing function with respect to k. Instead, it exhibits peak–valley characteristics like any resonance. It hits the maximum transmission near the quasi energy levels of the well between $-b$ and b. We illustrate this in the following under the assumptions that the barriers are strong such that

$$\cosh 2\kappa a \sim \sinh 2\kappa a \sim \exp(2\kappa a)/2 \tag{15a}$$

and the tunnelling is small for low energy such that

$$k \ll \kappa. \tag{15b}$$

The condition for the eigenvalues in the infinite well satisfies

$$\sin 2k_n b = 0 \qquad \text{or} \qquad k_n b = n\pi/2. \tag{16}$$

Substituting (15) and (16) into (5) we find

$$T_{\text{double}} = (-\mathrm{i}k/4\kappa)\exp(-4\kappa a)\exp(-4\mathrm{i}ka). \tag{17}$$

This simple estimate gives at least a factor of κ/k larger than other states not around the quasi levels near the bottom of the well. This is evidence of resonant tunnelling. The analysis above can be refined to include the shift from the quasi levels for the true resonance. Suppose the shift is small, we can expand $\exp(4\mathrm{i}kb)$ around $k_n b = n\pi/2$. This leads to

$$k_n b = n\pi/2 - n\pi/4\kappa_n b. \tag{18}$$

As a matter of fact, the peak will remain finite since the denominator of T_{double} from (5) does not vanish at any finite κa (although it could be rather small) and this will contribute to the natural width of the resonant tunnelling peak. In addition, there is a width associated with the various scattering mechanisms. We leave this for the numerical calculations presented later in this section.

In the double-barrier case, the interference effect is manifested in the enhanced tunnelling interference factor (ETIF) as shown in (10)

$$\text{ETIF} = \exp(\omega\tau) + \exp(-\mathrm{i}(\omega\tau' + \alpha)) \tag{19}$$

where ω and α are the parameters of oscillatory barriers defined in (2) and τ and τ' are defined in (8). When $\omega\tau' + \alpha = 2n\pi$, constructive interference occurs. On the other hand, $\omega\tau' + \alpha = (2\mathrm{n} + 1)\pi$ is the condition for destructive interference. Because the magnitude of ETIF can vary from $\exp(\omega\tau) - 1$ to $\exp(\omega\tau) + 1$, this is a profound effect for T_1 of coherent tunnelling especially when $\omega\tau$ is small. When α is random, the resulting tunnelling current is shown in (12), which is nothing other than the summation of two one-phonon-absorbed tunnelling currents by two individual barriers. Notice that the two currents are not identical: the first phonon-assisted tunnelling by the first barrier in (8) is larger than the second one in (9) (as it should be since the electron absorbing one phonon in the first barrier will maintain at E_1 in the second barrier). The overall picture is this: τ, the transversal time through the barrier (Büttiker and Landauer 1982) which is dependent on a but independent of b, determines what the extent of the enhanced tunnelling could be; τ' (which is dependent on b but independent of a) the real traversal time across the well, determines the interference of two one-phonon-absorbed states.

Finally, we present a numerical study of the phonon-assisted tunnelling and the interference effect in a model system described by Goldman *et al* (1987). The data for modelling GaAs–AlGaAs double-barriers are the following: the width of the GaAs well is $2b = 50$ Å, the width of the AlGaAs barrier is $2a = 80$ Å, and the barrier height is $V_0 = 0.4$ eV. The external potential V_1 reflects the interaction strength between the electron and the phonon. In the present semiconductor heterostructure case, it is mainly due to the longitudinal optic phonon (LO) (Goldman *et al* 1987, Wingreen *et al* 1988, Wendler 1985, Wendler and Pechstedt 1987, Jain and Das Sarma 1989). Stone *et al* (1985) treat electron–phonon scattering semiclassically as a weak perturbation. We take $V_1 = 4$ meV which is in accordance with their $\Gamma_0 = 0.1$. The frequency of the temporal oscillation is chosen to be that of the bulk LO modes. These modes exhibit a slight dispersion and we take $\hbar\omega = 300$ K $= 25$ meV (Wingreen *et al* 1988, Wendler 1985, Wendler and Pechstedt 1987, Jain and Das Sarma 1989).

The width of the tunnelling peak is associated with the scattering mechanism like any resonance phenomenon and it is determined by the magnitude of the hopping matrix element in the electronic Hamiltonian. The origin of the width can be attributed to electron–electron, electron–impurity, electron–defect and electron–acoustic phonon scattering, etc. The observation of sharp structures in the conductance suggests a narrow resonance width (compared to the energy level spacing) for both elastic and inelastic resonance. Actually, one group (Kopley *et al* 1988) obtained $\Gamma = 0.25$ meV. However, another group (Wingreen *et al* 1988) reported that the resonance width $\Gamma = 0.2\,\hbar\omega$, where ω is the LO frequency. This width is about 8 meV if $\hbar\omega = 40$ meV (Goldman *et al* 1987) in AlGaAs barriers, which certainly washes out all the most rugged features but maintains a coarse-grained resonance structure. Our numerical studies take $\Gamma = 5$ meV in agreement with Goldman *et al* (1987) and Wingreen *et al* (1988). We argue that the sharp rugged features in the tunnelling spectra could be due to the interference effect of two uncorrelated barrier phonon modes on top of the experimental noise.

The calculation is based on (10) with the resonance width Γ built in (5) through the following expression (Stone and Lee 1986):

$$T_{\mathrm{w}}(E) = T(E)\,\mathrm{i}\Gamma/2\pi/[(E - E_0) + \mathrm{i}\Gamma/2] \tag{20}$$

where E_0 is the energy at resonance.

Figure 2 shows the resonant peak of tunnelling probability $|T|^2$ at $E_0 = 185$ meV which is the fourth level† in the quantum well, the one-phonon-absorbed peak at $E_1 = E_0 + \hbar\omega = 210$ meV and a barely-observable one-phonon-emitted peak at $E_{-1} = E_0 - \hbar\omega = 160$ meV. The one-phonon-emitted tunnelling amplitude here is obtained by replacing $\omega\tau$ by $-\omega\tau$ in (10). The two tunnelling sidebands are quite asymmetric

† The reason in choosing this resonant state is because it lies in the middle of the barrier height in order to maintain the validity of the low-frequency approximation in (7).

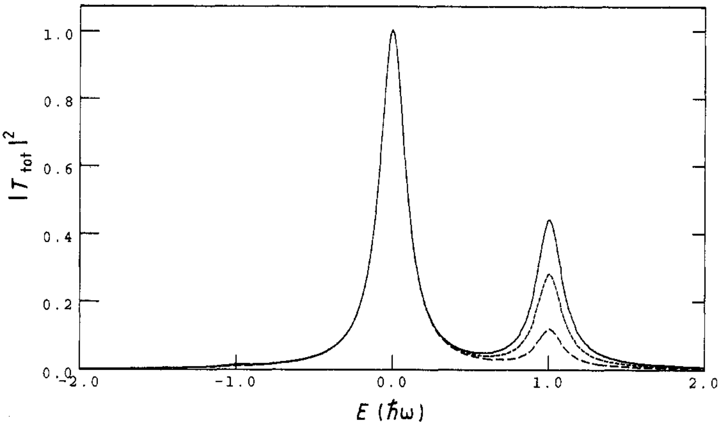

Figure 2. The transmission probability versus the energy of the electrons in units of phonon energy, $\hbar\omega$. The zero of the energy is in the fourth resonant state in the quantum well. The full curve stands for the constructive tunnelling, the long-dashed curve for the destructive case, and the short-dashed curve represents the incoherent tunnelling.

because of the different dynamical factors $\exp(\omega\tau)$ and $\exp(-\omega\tau)$. The phonon-assisted-tunnelling peaks also exhibit the resonance behaviour which is responsible for the low resolution, i.e. large energy scale structure of the conductance. Furthermore, the coherence plays an important role: the in-phase oscillation of double barriers gives the constructive interference for enhanced tunnelling probability amounting to 45% of the principal peak in the present model study, the out-of-phase oscillation gives the destructive interference which amounts to 10%, and the random-phase oscillation gives half of the constructive interference which is about 28%. The suppressed tunnelling probability is about 1.5% which can be barely recognised in figure 2. This is of the same order as Goldman *et al* (1987) which reported a value of 4% of the principal peak. However, it is apparently ten times smaller in comparison to the calculation of Wingreen *et al* (1988). This is probably due to our small perturbation parameter V_1 in addition to the much smaller dynamical factor $\exp(-\omega\tau)$ in the phonon-emitted tunnelling. The incoherence case is the one observed in the experiment where the thermal state of the two slab phonon modes is not phase coherent. The curve representing the incoherent case should be understood as the ensemble average which is done by repeating the current measurements a large number of times. If the experiment is conducted without this extra averaging, one actually observed the result from individual phase differences. Since the sweeping frequency measurements are usually not done *in situ*, it leads to the stochastic phase difference and introduces the sharp, fine features of the spectra, i.e. the fluctuating conductance in the small energy scale. However, the detection of the tunnelling current takes quite a long time and it effectively takes the average process of the different individual phases since the measuring time is usually much longer than the coherence time of the thermal phonon bath. We believe that this extra degree of complexity explains the rugged I–V curve.

3. Discussion

We have shown in this work the importance of phonon-assisted tunnelling and its resonance and coherence aspects. This helps our understanding of complicated features of the tunnelling spectra of the double barriers. It shows that the qualiative conclusion for the localised phonon modes remains valid for the extended modes and the sharp rugged feature can be attributed to the resonant tunnelling and the resonant phonon-assisted tunnelling as well as the sampling of the uncorrelated phonon modes that exist in the two-barrier regions. Moreover this study opens the way in studying the coherent tunnelling spectra.

The nature of the electron–phonon interaction and the slab phonon in the heterostructure is crucial in understanding its tunnelling spectra. First of all, the primary contribution to the electron–phonon coupling is the longitudinal optic (LO) phonons (polarisation eigenmodes) since the series III–V semiconductor materials are strongly polar-like and its longitudinal part of the electric field is coupled to the charge of the electron. This paper only treats the bulk LO phonon in the two-barrier regions, however, the LO phonon in the quantum well and the interfacial LO modes are also important in principle. There are also indications (Wingreen *et al* 1988, Wendler 1985, Wendler and Pechstedt 1987, Jain and Das Sarma 1989) that the interfacial phonon mode are important in the problem of hot electron relaxation in quantum wells. The accurate solution of the phonon modes is a formidable task since the GaAs and GaAlAs layers result in a complicated boundary geometry. So far, all the theoretical calculations assume that the polarisation waves are completely backscattered from the interface. This results in the confinement of the slab phonon modes either to the barrier or to the well regions. If this is the case, it prohibits the forward propagation of LO phonons through the interface, thus, preventing any interference of LO phcnons within two barriers. In order to observe the interesting constructive or destructive interference of tunnelling through double barriers, either the breakdown of the confinement approximation or some clever control of the phases of the excited phonon modes in different regions is required. In the former case, the phase difference α is determined, in principle, by the thickness and the dispersion relation of the material in the well if the transmitted phonon is considered. The coherent lattice motion might be excited by the coherent laser pulse like the excited molecular vibrational modes. We have not included in our work the phonon modes inside the well. However, our two-potential formula (4) is still valid of these modes are included and the enhancement due to these modes are also expected. The numerical example only considers the bare electron mass, we will address these questions in future work.

As to the convergence of the perturbation series, we want to emphasise that the small parameter is not only $V_1/2\hbar\omega$ which is 0.08 in our numerical study. In fact, the small parameter is $V_1/2\hbar\omega(\exp(\omega\tau) - 1)$ which is about 0.64. It reaches $V_1\tau/2\hbar$ in the small-ω limit. It seems that this parameter might be too big to breakdown the perturbation expansion. However, for the single-barrier case we have calculated up to the second-order perturbation and the summation is more like the expansion of the exponential function.† If this is the case, convergence is guaranteed for any parameter. Nevertheless, the difficulties are now mathematical as regards the derivation of the *n*th order perturbation of the single-barrier as well as the double-barrier model. More work will be undertaken in this direction.

† We solved the two-quanta absorption case using BL's method (Jiang 1989). The solution in the low-frequency approximation is $T_2 \exp(\mathrm{i}k_2 a) = T_0 \exp(\mathrm{i}ka)(V_1^2/8\hbar^2\omega^2)(\exp(\omega\tau) - 1)^2$. Apparently, it is consistent with the definition of the traversal time of tunnelling since $T_2 \exp(\mathrm{i}k_2 a) = T_0 \exp(\mathrm{i}ka)(V_1^2\tau^2/2\hbar^2)$ when $\omega\tau \ll 1$ and $\tau^2/2 = \int_0^\tau \mathrm{d}t_2 \int_0^{t_2} \mathrm{d}t_1$. One may conjecture that $T_n \exp(\mathrm{i}k_n a) = T_0 \exp(\mathrm{i}ka)(V_1^n\tau^n/n!\hbar^n)$ holds for n-quanta absorption.

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